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Supersymmetric AdS_3, AdS_2 and bubble solutions

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Abstract: We present new supersymmetric AdS_3 solutions of type IIB supergravity and AdS_2 solutions of $D = 11$ supergravity. The former are dual to conformal field theories in two dimensions with $N = (0, 2)$ supersymmetry while the latter are dual to conformal quantum mechanics with two supercharges. Our construction also includes AdS_2 solutions of $D = 11$ supergravity that have non-compact internal spaces which are dual to three-dimensional $N = 2$ superconformal field theories coupled to point-like defects. We also present some new bubble-type solutions, corresponding to BPS states in conformal theories, that preserve four supersymmetries.

Keywords: AdS-CFT Correspondence, Supergravity Models.

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Contents

1. Introduction

Supersymmetric solutions of $D = 10$ and $D = 11$ supergravity that contain AdS factors are dual to superconformal field theories (SCFTs). It is therefore of interest to study the generic geometric structure of such solutions and, in particular, to use this insight to construct new explicit solutions.

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The most general supersymmetric solutions of type IIB supergravity with an AdS_3 factor and with only the five-form flux non-trivial were analysed in [1]. These solutions can arise as the near-horizon geometry of configurations of D3-branes, preserve 1/8-th of the supersymmetry and are dual to two-dimensional $N = (0, 2)$ superconformal field theories. Similarly, in [2] the most general supersymmetric solutions of $D = 11$ supergravity with an AdS_2 factor and a purely electric four-form flux were analysed. These solutions can arise as the near-horizon geometry of configurations of M2-branes, also preserve 1/8-th of the supersymmetry and are dual to superconformal quantum mechanics with two supercharges.

We shall summarise the main results of [1, 2] in section 2 below. What is remarkable is that the internal manifolds in each case have the same geometrical structure. For the type IIB AdS_3 solutions, locally the seven-dimensional internal manifold Y_7 has a natural foliation, such that the metric is completely determined by a Kähler metric on the six-dimensional leaves. For the $D = 11$ AdS_2 solutions, locally the metric on the internal manifold Y_9 is again completely determined by a Kähler metric on, now, eight-dimensional leaves. Both $(2n+2)$-dimensional Kähler metrics ds^2_{2n+2} satisfy exactly the same differential condition

$$\Box R - \tfrac{1}{2}R^2 + R_{ij}R^{ij} = 0 \tag{1.1}$$

where R and R_{ij} are the Ricci-scalar and Ricci-tensor, respectively, of the metric ds^2_{2n+2}. In each case, to obtain an AdS_3 or AdS_2 solution one requires $R > 0$.

It is worth noting the similarities with Sasaki-Einstein (SE) metrics. Recall that five-dimensional SE metrics, SE_5 give rise to supersymmetric solutions of type IIB supergravity of the form $AdS_5 \times SE_5$, while seven-dimensional SE metrics, SE_7 give rise to supersymmetric solutions of $D = 11$ supergravity of the form $AdS_4 \times SE_7$. All SE metrics have a canonical Killing vector which defines, at least locally, a canonical foliation, and the SE metric is completely determined by a Kähler-Einstein metric on the corresponding leaves. There has been some recent explicit constructions of local Kähler-Einstein metrics that give rise to complete SE metrics and we will show that they can be adapted to produce Kähler metrics that satisfy (1.1) and hence give rise to new AdS_3 and AdS_2 solutions.

After an analytic continuation, the generic AdS_3 and AdS_2 solutions discussed in [1, 2] give rise to generic supersymmetric solutions with S^3 and S^2 factors preserving 1/8-th of maximal supersymmetry. In particular the solutions are built from the same Kähler geometry satisfying (1.1), but now with $R < 0$. Such "bubble solutions" generalise the 1/2 supersymmetric bubble solutions of [3] (1/4 supersymmetric bubble solutions in type IIB were analysed in [4, 5]) and generically have an $\mathbb{R} \times \mathrm{SO}(4)$ or $\mathbb{R} \times \mathrm{SO}(3)$ group of isometries. Depending on the boundary conditions, the 1/8-th supersymmetric bubbles can describe 1/8-th BPS states in the maximally SCFTs, or other BPS states in SCFTs with less supersymmetry. Note that recently an analysis of 1/8-th supersymmetric bubbles in type IIB supergravity with additional symmetries was carried out in [6] and, most recently, AdS_2 and bubble solutions of $D = 11$ supergravity preserving various amounts of supersymmetry were analysed in [7]. The constructions that we use for the AdS_3 and AdS_2 solutions, that we outline below, also lead to new explicit bubble solutions.

We will present three constructions of Kähler metrics satisfying (1.1) which lead to

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new AdS and bubble solutions. The first construction is directly inspired by the constructions of SE metrics in [8, 9]. Following [10], the idea is to build the local Kähler metric from a $2n$-dimensional Kähler-Einstein metric $ds^2(KE_{2n})$. To construct complete metrics on the internal space Y_{2n+3} we will usually assume that the $2n$-dimensional leaves on which $ds^2(KE_{2n})$ is defined extend to form a compact Kähler-Einstein space KE_{2n}. One might then try to similarly extend the Kähler metrics ds^2_{2n+2} to give non-singular metrics on a compact space which is an S^2 fibration over KE_{2n}. However, this is not possible. Nonetheless, as we show, this kind of construction can give rise to complete and compact metrics on Y_{2n+3}. This is precisely analogous to the construction of Sasaki-Einstein manifolds presented in [8, 9]. For the six-dimensional case, we show that we recover the AdS_3 solutions of type IIB supergravity that were recently constructed in [11]. On the other hand, the eight-dimensional case leads to new infinite classes of AdS_2 solutions of $D = 11$ supergravity.

In [11] it was shown that by choosing the range of coordinates differently, one obtains AdS_3 solutions with non-compact internal spaces. These solutions were interpreted as being dual to four-dimensional $\mathcal{N} = 1$ SCFTs, arising from five-dimensional Sasaki-Einstein spaces, in the presence of a one-dimensional defect. Similarly, we can also choose the range of the coordinates in the new AdS_2 solutions presented here so that they also have non-compact internal spaces. As we discuss, these solutions are dual to three-dimensional $\mathcal{N} = 2$ SCFTs arising from seven-dimensional Sasaki-Einstein spaces, in the presence of a point-like defect.

We will then show that this first construction of Kähler metrics also gives rise to supersymmetric bubble solutions. Indeed, remarkably, we find that we recover the uplifted versions of the AdS single-charged "black hole" solutions of minimal gauged supergravity in $D = 5$ and $D = 4$. Recall that these BPS solutions have naked singularities and hence were christened "superstars" in [12, 13]. These solutions are special cases of more general superstar solutions obtained by uplifting three- and four-charged AdS "black holes" in $D = 5$ and $D = 4$ gauged supergravity, respectively. We identify the underlying Kähler geometry for these general solutions which we then employ to carry out our second construction of supersymmetric $AdS3$ and AdS_2 solutions. The Kähler geometries are toric and hence this construction is analogous to the construction of SE metrics of [14] (see also [15]).

The third construction of Kähler metrics satisfying (1.1) that we shall present is to simply take the metric to be a direct product of Kähler-Einstein metrics. This gives rise to rich new classes of AdS_3 and AdS_2 solutions. A special case of the AdS_3 solutions is that given in [16], describing $D3$-branes wrapping a holomorphic curve in a Calabi-Yau four-fold, while a special case of the AdS_2 solutions corresponds to the solution in [17], describing $M2$-branes wrapping a holomorphic curve in a Calabi-Yau five-fold. The construction also gives rise to infinite new bubble solutions.

The plan of the rest of the paper is as follows. We begin in section 2 by reviewing the construction of [1, 2]. In section 3 we describe the construction using S^2 fibrations over KE manifolds. In section 4 we show that this construction gives rise to bubble solutions that are the same as the uplifted single charged superstars. In section 5, we determine the Kähler geometry underlying the multiple charged superstars and then use this to construct

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infinite new class of AdS solutions. In section 6 we describe the construction of AdS and bubble solutions using products of KE metrics. Section 7 briefly concludes.

2. Background

2.1 AdS_3 in IIB and AdS_2 in $D = 11$

The generic AdS_3 and AdS_2 solutions discussed in [1, 2] are constructed as follows. In each case, one assumes the metric is a warped product

$$ds^2 = L^2 e^{2A} \left[ds^2(AdS_d) + ds^2(Y_{2n+3})\right] \tag{2.1}$$

where we normalise such that $ds^2(AdS_d)$ has unit radius and L is an overall scale factor that we will sometimes normalise to one. Let ds^2_{2n+2} be a $2n+2$-dimensional Kähler metric satisfying (1.1).

The generic 1/8-th supersymmetric AdS_3 solution of type IIB supergravity with non-trivial five-form is then given by taking the metric on Y_7 to have the form [1]

$$ds^2(Y_7) = \tfrac{1}{4}(dz + P)^2 + e^{-4A} ds_6^2 \tag{2.2}$$

where $dP = \mathcal{R}$ (the Ricci form on ds_6^2). The warp factor is given by $e^{-4A} = \frac{1}{8}R$ and hence we must have $R > 0$. The five-form flux is given by

$$F_5 = L^4 (1 + *) \,\mathrm{vol}(AdS_3) \wedge F \tag{2.3}$$

with

$$F = \tfrac{1}{2} J - \tfrac{1}{8} d\left[e^{4A}(dz + P)\right] \tag{2.4}$$

Using the fact that the Ricci-form of the Kähler metric ds^2_{2n+2} satisfies

$$*_{2n+2}\mathcal{R} = \frac{R}{2}\frac{J^n}{n!} - \frac{J^{n-1}}{(n-1)!} \wedge \mathcal{R} \tag{2.5}$$

we can rewrite the five-form flux as

$$F_5 = L^4 \,\mathrm{vol}(AdS_3) \wedge F + \frac{L^4}{16}\left[J \wedge \mathcal{R} \wedge (dz + P) + \frac{1}{2} *_6 dR\right] \tag{2.6}$$

since F is clearly closed, we see that F_5 is closed as a result of the condition (1.1). The vector ∂_z is Killing and preserves the five-form flux. The solutions are dual to two-dimensional conformal field theories with $(0, 2)$ supersymmetry. Since only the five-form flux is non-trivial, solutions of this type can be interpreted as arising from the back-reacted configurations of wrapped or intersecting D3-branes. For example, we shall show that there are such solutions that correspond to D3-branes wrapping holomorphic curves in Calabi-Yau four-folds.

The generic 1/8-th supersymmetric AdS_2 solution of $D = 11$ supergravity with purely electric four-form flux is given by taking the internal metric [2]

$$ds^2(Y_9) = (dz + P)^2 + e^{-3A} ds_8^2 \tag{2.7}$$

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with $dP = \mathcal{R}$. The warp factor is $e^{-3A} = \frac{1}{2}R$ and so again we must take $R > 0$. The four-form flux is given by

$$G_4 = L^3 \,\mathrm{vol}(AdS_2) \wedge F \tag{2.8}$$

with

$$F = -J + d\left[e^{3A}(dz + P)\right] \tag{2.9}$$

The four-form G_4 is clearly closed. Using (2.5) we have

$$*G_4 = \frac{J^2}{2} \wedge \mathcal{R} \wedge (dz + P) + \frac{1}{2} *_8 dR \tag{2.10}$$

and hence the equation of motion for the four-form, $d * G_4 = 0$, is satisfied as a result of (1.1). Again, the vector ∂_z is Killing and preserves the flux. These solutions are dual to conformal quantum mechanics with two supercharges. The fact that the four-form flux is purely electric means such solutions can be interpreted as arising from the back-reacted configurations of wrapped or intersecting M2-branes. For example, as we will see, there are such solutions that correspond to M2-branes wrapping holomorphic curves in Calabi-Yau five-folds.

Note for both cases that if we scale the Kähler metric by a positive constant it just leads to a scaling of the overall scale L of the $D = 10$ and $D = 11$ solutions.

Finally, we note that a particular class of the $D = 11$ solutions can be related to the type IIB solutions. Suppose that there is a pair of commuting isometries of the Kähler metric ds_8^2 such that globally they parametrise a torus T^2 and the nine-dimensional internal manifold is metrically a product $T^2 \times M_7$. By dimensional reduction and T-dualising on this T^2 we can obtain a type IIB solution with an AdS_2 factor. In fact, as we show in the appendix, one actually obtains a type IIB solution with an AdS_3 factor, precisely of the form (2.2)–(2.4).

2.2 Bubble solutions

The AdS solutions discussed thus can in general be analytically continued to describe stationary geometries with S^3 and S^2 factors in type IIB and $D = 11$ supergravity respectively. Such "bubble solutions" again preserve 1/8-th of the maximal supersymmetry and generalise the 1/2 supersymmetric bubble solutions of [3]. Generically they have an $\mathbb{R} \times \mathrm{SO}(4)$ or $\mathbb{R} \times \mathrm{SO}(3)$ group of isometries. Depending on the boundary conditions, they can correspond to 1/8-th BPS states in the maximally SCFTs, or other BPS states in SCFTs with less supersymmetry.

To obtain the type IIB bubble solutions one adapts the analysis of [1], by replacing the AdS_3 factor with an S^3. The local form of the metric is given by

$$ds^2 = L^2 e^{2A}\left[-\frac{1}{4}(dt + P)^2 + ds^2(S^3) + e^{-4A} ds_6^2\right] \tag{2.11}$$

where ds_6^2 is, as before, a Kähler metric with Ricci form $\mathcal{R} = dP$ satisfying (1.1). Note that now we have a time-like Killing vector ∂_t. The warp factor is given by $e^{-4A} = -\frac{1}{8}R$ and so now we want solutions with $R < 0$. The five-form flux becomes

$$F_5 = L^4(1 + *)\,\mathrm{vol}(S^3) \wedge F \tag{2.12}$$

with

$$F = \tfrac{1}{2}J + \tfrac{1}{8}d\left[e^{4A}(dt+P)\right] \tag{2.13}$$

The Killing vector ∂_t also preserves the five-form flux.

Similarly adapting the analysis of [2] by replacing the AdS_2 factor by S^2 one can construct new bubbling 1/8-th supersymmetric solutions of $D = 11$ supergravity. The local form of the metric becomes

$$ds^2 = L^2 e^{2A}\left[-(dt+P)^2 + ds^2(S^2) + e^{-3A}ds_8^2\right] \tag{2.14}$$

with $dP = \mathcal{R}$ and the Ricci-form again satisfies (1.1). The warp factor is now $e^{-3A} = -\frac{1}{2}R$ and so we again want solutions with $R < 0$. The four-form flux is given by

$$G_4 = L^3\,\mathrm{vol}(S^2)\wedge F \tag{2.15}$$

with

$$F = -J - d\left[e^{3A}(dt+P)\right] \tag{2.16}$$

and again it is preserved by the Killing vector ∂_t.

3. Fibration constructions using KE^+_{2n} spaces

In order to find explicit examples of Kähler metrics in $2n+2$ dimensions satisfying (1.1), we follow [10] and also [8, 9], and consider the ansatz

$$ds^2_{2n+2} = \frac{d\rho^2}{U} + U\rho^2(D\phi)^2 + \rho^2 ds^2(KE^+_{2n}) \tag{3.1}$$

with

$$D\phi = d\phi + B \tag{3.2}$$

Here $ds^2(KE^+_{2n})$ is a $2n$-dimensional Kahler-Einstein metric of positive curvature. It is normalised so that $\mathcal{R}_{\rm KE} = 2(n+1)J_{\rm KE}$ and the one-form form B satisfies $dB = 2J_{\rm KE}$. Note that $(n+1)B$ is then the connection on the canonical bundle of the Kähler-Einstein space. Let $\Omega_{\rm KE}$ denote a local $(n,0)$-form, unique up rescaling by a complex function.

To show that ds^2_{2n+2} is a Kähler metric observe that the Kähler form, defined by

$$J = \rho d\rho\wedge D\phi + \rho^2 J_{\rm KE}, \tag{3.3}$$

is closed, and that the holomorphic $(n+1,0)$-form

$$\Omega = e^{i(n+1)\phi}\left(\frac{d\rho}{\sqrt{U}} + i\rho\sqrt{U}D\phi\right)\wedge\rho^n\Omega_{\rm KE} \tag{3.4}$$

satisfies

$$d\Omega = if D\phi\wedge\Omega \tag{3.5}$$

with

$$f = (n+1)(1-U) - \frac{\rho}{2}\frac{dU}{d\rho} \tag{3.6}$$

This implies, in particular, that the complex structure defined by Ω is integrable. In addition (3.5) allows us to obtain the Ricci tensor of ds^2_{2n+2}:

$$\mathcal{R} = dP, \qquad P = fD\phi \tag{3.7}$$

The Ricci-scalar is then obtained via $R = \mathcal{R}_{ij}J^{ij}$.

We would like to find the conditions on U such that ds^2_{2n+2} satisfies the equation (1.1). It is convenient to introduce the new coordinate $x = 1/\rho^2$ so that

$$ds^2_{2n+2} = \frac{1}{x}\left[\frac{dx^2}{4x^2U} + U(D\phi)^2 + ds^2(KE^+_{2n})\right] \tag{3.8}$$

and

$$f = (n+1)(1-U) + x\frac{dU}{dx} \tag{3.9}$$

$$R = 4nxf - 4x^2\frac{df}{dx} \tag{3.10}$$

We can now show that (1.1) can be integrated once to give

$$2nf^2 + U\frac{dR}{dx} = Cx^{n-1} \tag{3.11}$$

where C is a constant of integration.

For simplicity, in what follows, we will only consider polynomial solutions of (3.11). In particular, if $U(x)$ is a k-th order polynomial we have the following indicial equation: $(k-n-1)(k-n+1)(2k-n) = 0$, which implies that $k = n+1$. Thus our problem is to find polynomials of the form

$$U(x) = \sum_{j=0}^{n+1} a_j x^j \tag{3.12}$$

satisfying (3.11). Note from (3.9) that the Ricci scalar is given by

$$R = 4x\left[n(n+1) - \sum_{j=0}^{n-1}(n-j)(n-j+1)a_j x^j\right] \tag{3.13}$$

Since R is related to the AdS warp factor, for a consistent warped product we see that the range of x must exclude $x = 0$. Furthermore, from (3.8) we must take $x > 0$ and $U(x) > 0$.

Our main interest is the six-dimensional ($n = 2$) and eight-dimensional ($n = 3$) cases, which give rise to type IIB and $D = 11$ solutions respectively. If $n = 2$, the function $U(x)$ is cubic and the condition (3.11) implies that

$$\begin{aligned} a_2^2 - 4a_3a_1 &= 0 \\ a_3(1-a_0) &= 0 \\ (a_0-1)(a_0-3) &= 0 \end{aligned} \tag{3.14}$$

The Ricci scalar is given by

$$R = 8x\,(3 - 3a_0 - a_1x) \tag{3.15}$$

We have two choices depending on the solution of the second equation of (3.14). The first is $a_3 = 0$ and hence $a_2 = 0$ with

$$U(x) = a_0 + a_1 x \tag{3.16}$$

and either $a_0 = 1$ or $a_0 = 3$. The second choice is $a_3 \neq 0$, $a_0 = 1$ and hence $a_2^2 = 4a_3 a_1$ with

$$U(x) = 1 + a_1 x + a_2 x^2 + (a_2^2/4a_1)x^3 \tag{3.17}$$

(note that when $a_1 = a_2 = 0$ we have $R = 0$ and so we ignore this case.)

If $n = 3$ the function $U(x)$ is quartic and the condition (3.11) implies that

$$\begin{aligned} a_3^2 - 4a_4 a_2 &= 0 \\ a_4 a_1 &= 0 \\ a_1(2 - a_0) &= 0 \\ a_3 a_1 - 4a_4(1 - a_0) &= 0 \\ (a_0 - 1)(a_0 - 2) &= 0 \end{aligned} \tag{3.18}$$

The Ricci scalar is given by

$$R = 8x\left(6 - 6a_0 - 3a_1 x - a_2 x^2\right) \tag{3.19}$$

Solving the equations (3.18) again leads to two classes of solutions depending on the solution of the second equation. First we take $a_4 = 0$ which implies $a_3 = 0$ with

$$U(x) = a_0 + a_1 x + a_2 x^2 \tag{3.20}$$

and either $a_0 = 1$, $a_1 = 0$ or $a_0 = 2$. Alternatively we take $a_4 \neq 0$, $a_1 = 0$ and hence $a_0 = 1$ and $a_3^2 = 4a_4 a_2$ with

$$U(x) = 1 + a_2 x^2 + a_3 x^3 + (a_3^2/4a_2)x^4 \tag{3.21}$$

(note that when $a_0 = 1, a_1 = a_2 = 0$ we have $R = 0$ and so we ignore this case.)

For the remainder of this section we will only consider AdS solutions ($R > 0$), returning to bubble solutions ($R < 0$) in the next section. In order that the AdS solutions are globally defined, in the following we will usually assume that the local leaves in (3.8) with metric $ds^2(KE_{2n}^+)$ extend globally to form a compact Kähler-Einstein manifold KE_{2n}^+ and that the internal manifold Y_{2n+3} in (2.1) is a fibration over KE_{2n}^+. We could also assume that x and ϕ in (3.8) separately describe a fibration over KE_{2n}^+. In particular, if the range of x is taken to lie between two zeroes of $U(x)$, then, at a fixed point on KE_{2n}^+, (x, ϕ) can parametrise a two-sphere ($U(x)$ has to have a suitable behaviour at the zeroes to avoid conical singularities). Topologically this can then form a two-sphere bundle over KE_{2n}^+ which is just the canonical line bundle KE_{2n}^+ with a point "at infinity" added to each of the fibres. In fact we shall see that in the solutions we discuss this possibility is *not* realised and that the metric necessarily has conical singularities at one of the poles of the two-sphere. However, as we shall also see, in the $D = 10$ and $D = 11$ supergravity solutions, after adding in the extra z-direction, in the resulting spaces Y_{2n+3} two-sphere

bundles over KE^+_{2n} (without conical singularities) do appear but where the polar angle on the sphere is a combination of ϕ and z.

We will now discuss the six-dimensional ($n = 2$) and eight-dimensional ($n = 3$) cases in turn, corresponding to type IIB AdS_3 and $D = 11$ AdS_2 solutions respectively.

3.1 Fibrations over KE^+_4: type IIB AdS_3 solutions

For these solutions, the warp factor is given by $R = 8e^{-4A}$ which must be positive. Recall that we had two choices for the function $U(x)$. First consider the case $U(x) = a_0 + a_1 x$ with either $a_0 = 1$ or $a_0 = 3$. From (3.8) for a compact Y_7 manifold with finite warp factor we need a finite range of $x > 0$ between two solutions of $U(x) = 0$, such that $U(x) > 0$ (so that we have the right signature). Since $U(x)$ is linear for this case, it has only one root and so there are no compact solutions. (In fact, as we discuss later, the case where $a_0 = 1$ corresponds to $AdS_5 \times X_5$, where X_5 is a Sasaki-Einstein manifold.)

We thus consider the second case $U(x) = 1 + a_1 x + a_2 x^2 + (a_2^2/4a_1)x^3$. We now show that this gives rise to the family of type IIB AdS_3 solutions found in [11]. To compare with the solutions given in [11] we need to make a number of transformations. First it is convenient to change parametrization and write $a_3 = -1/\alpha^3, a_2 = 2\beta/\alpha^3, a_1 = -\beta^2/\alpha^3$ so that

$$U(x) = 1 - \frac{x(x-\beta)^2}{\alpha^3}. \tag{3.22}$$

The scalar curvature is given by

$$R = \frac{8\beta^2}{\alpha^3} x^2 \tag{3.23}$$

and we must choose $\alpha > 0$ to ensure that $R > 0$. The metric (2.2) on the internal manifold is then given by

$$ds^2(Y_7) = \frac{1}{4}\left[dz - \frac{2\beta x(x-\beta)}{\alpha^3} D\phi\right]^2 + \frac{\beta^2}{\alpha^3}\left[\frac{dx^2}{4xU} + xU(D\phi)^2 + x ds^2(KE^+_4)\right] \tag{3.24}$$

Note that this is invariant under simultaneous rescalings of x, α and β. Using this symmetry we can set

$$\beta = \frac{4}{3a}, \qquad \alpha^3 = \frac{256}{729a^2} \tag{3.25}$$

Introducing new coordinates $y = 4/(9x)$ and $\psi = 3\phi + z$ we can rewrite the metric as

$$ds^2(Y_7) = \frac{y^2 - 2y + a}{4y^2} Dz^2 + \frac{9dy^2}{4q(y)} + \frac{q(y) D\psi^2}{16y^2(y^2 - 2y + a)} + \frac{9}{4y} ds^2(KE^+_4) \tag{3.26}$$

where $D\psi = d\psi + 3B$, $Dz = dz - g(y)D\psi$ and

$$\begin{aligned} q(y) &= 4y^3 - 9y^2 + 6ay - a^2 \\ g(y) &= \frac{a - y}{2(y^2 - 2y + a)} \end{aligned} \tag{3.27}$$

The warp factor is simply $e^{2A} = y$. Using (2.3) and (2.4), we find the five-form flux is

$$F = -\frac{1}{4} y dy \wedge dz + \frac{3a}{8} J_{\mathrm{KE}} \tag{3.28}$$

This can now be directly compared with the solution constructed in [11]. We need to take into account the different normalization conventions for the KE_4^+. This requires rescaling $ds^2(KE_4^+)$ by a factor of 1/6. Recalling that by definition $3B$ is the connection on the canonical bundle of KE_4^+, we see that (3.26) agrees precisely with the metric given in [11]. Furthermore, the expression for the five-form also agrees, again allowing for a difference in conventions: the five form being used here is $-1/4$ that of [11].

The regularity of these solutions were discussed in detail in [11]. Restricting y to lie between the two smallest roots of the cubic $q(y)$, topologically the solutions as written in (3.26) are $U(1)$ bundles, with fibre parametrised by z, over an S^2 bundle, with fibre parametrised by (ψ, y), over KE_4^+. Note also, as was mentioned above, that the six-dimensional Kähler leaves parametrised by (x, ϕ) and KE_4^+ appearing in (3.24) are not S^2 bundles over KE_4^+ as there is necessarily a conical singularity at one of the poles.

3.2 Fibrations over KE_6^+: $D = 11$, AdS_2 solutions

We now discuss the case where $n = 3$ and Y_9 is nine-dimensional and look for AdS_2 solutions to $D = 11$ supergravity. For these solution the warp factor is given by $R = 2e^{-3A}$. Recall that there were two choices for the function $U(x)$. First consider $U(x) = a_0 + a_1 x + a_2 x^2$. Recall again that for a compact Y_9 geometry with finite warp factor we need to find a range of $x > 0$ between the two roots of $U(x) = 0$ over which $U(x) > 0$. Since $a_0 = 1$ or $a_0 = 2$, this is not possible and thus there are no compact warped product solutions in this class. (In fact, as we discuss later, the case $a_0 = 1$ corresponds to $AdS_4 \times X_7$ where X_7 is Sasaki-Einstein.)

We thus focus on the second case for which $U(x) = 1 + a_2 x^2 + a_3 x^3 + (a_3^2/4a_2)x^4$. This implies $R = -8a_2 x^3$. Since we require $x > 0$, $R > 0$ we must have $a_2 < 0$ and hence $a_4 < 0$. It is then useful to redefine $a_4 = -1/\alpha^4$, $a_3 = 4\beta/\alpha^4$ and $a_2 = -4\beta^2/\alpha^4$, with $\alpha > 0$, so that

$$U(x) = 1 - x^2(x - 2\beta)^2/\alpha^4 \tag{3.29}$$

and

$$R = \frac{32\beta^2}{\alpha^4} x^3 \tag{3.30}$$

The metric on the internal space Y_9 is then given by

$$ds^2(Y_9) = (dz + P)^2 + \frac{16\beta^2}{\alpha^4}\left[\frac{dx^2}{4U} + x^2 U (D\phi)^2 + x^2 ds^2(KE_6^+)\right] \tag{3.31}$$

where $P = -4\alpha^{-4}\beta x^2(x - 2\beta)D\phi$. Note that the metric is invariant under simultaneous rescalings of x, β and α. The roots of $U(x) = 0$ are given by

$$\begin{aligned} x_1 &= \beta - \sqrt{\beta^2 + \alpha^2} \qquad & x_2 &= \beta - \sqrt{\beta^2 - \alpha^2} \\ x_3 &= \beta + \sqrt{\beta^2 - \alpha^2} \qquad & x_4 &= \beta + \sqrt{\beta^2 + \alpha^2} \end{aligned} \tag{3.32}$$

Note that for $\beta^2 > \alpha^2$ we have four real roots and $U(x) \geq 0$ for $x \in [x_1, x_2]$ and $x \in [x_3, x_4]$. Demanding that $x > 0$, $R > 0$ we deduce that $\beta^2 > \alpha^2$, $\beta > 0$ and $x \in [x_3, x_4]$.

As in the type IIB case, it is not possible to avoid conical singularities at both x_3 and x_4 just by adjusting the period of ϕ. However, we can take an appropriate linear combination of z and ϕ and find a smooth compact manifold. To this end, we change coordinates

$$\psi = 4\phi + z \tag{3.33}$$

It is also convenient to use the scaling symmetry to set

$$\beta = \frac{3^{3/2}}{2^{11/2}a}, \qquad \alpha^4 = \frac{3^6}{2^{22}a^2} \tag{3.34}$$

and change variables from x to $y = 3^{3/2}/(2^{11/2}x)$. In these coordinates the warp factor is simply

$$e^{2A} = \frac{2}{3}y^2. \tag{3.35}$$

With these changes the metric takes the form

$$ds^2(Y_9) = \frac{y^3 - 3y + 2a}{y^3}Dz^2 + \frac{4dy^2}{q(y)} + \frac{q(y)(D\psi)^2}{y^3(y^3 - 3y + 2a)} + \frac{16}{y^2}ds^2(KE_6^+) \tag{3.36}$$

where $D\psi = d\psi + 4B$, $Dz = dz - g(y)D\psi$ and

$$\begin{aligned} q(y) &= y^4 - 4y^2 + 4ay - a^2 \\ g(y) &= \frac{a - y}{y^3 - 3y + 2a}. \end{aligned} \tag{3.37}$$

The conditions $\beta > 0$ and $\beta^2 > \alpha^2$ translate into $0 < a < 1$. The function $U(x)$ has been replaced by $q(y)$, which again has four roots $y_1 < 0 < y_2 < y_3 < y_4$, for this range of a. The condition that $x \in [x_3, x_4]$ translates into $y \in [y_2, y_3]$.

Near a root $y = y_i$ we find that the (y, ψ) part of the metric is given by

$$\frac{16}{q'(y_i)}\left[dr^2 + \frac{q'(y_i)^2}{16y^3(y^3 - 3y - 2a)}(D\psi)^2\right] = \frac{16}{q'(y_i)}\left[dr^2 + r^2(D\psi)^2\right] \tag{3.38}$$

where $y - y_i = r^2$. Thus, remarkably, by choosing the period of ψ to be 2π we can avoid conical singularities at both $y = y_2$ and $y = y_3$. As a consequence we can look for solutions where the topology of Y_9 is a $U(1)$ bundle, whose fibre is parametrised by z, over an eight-dimensional manifold which is topologically a two-sphere bundle, parametrised by (y, ψ), over KE_6^+. Furthermore, since by definition $4B$ is the connection of the canonical bundle of KE_6^+, the two-sphere bundle is simply the canonical line bundle of KE_6^+ with a "point at infinity" added to each fibre.

In order to check that the $U(1)$ fibration, with fibre parametrised by z, is globally defined, we need ensure that the periods of $d(gD\psi)$ over all 2-cycles of the eight-dimensional base space are integer valued. The problem is very similar to the type IIB solutions and we can follow the analysis of [11]. If we let the period of z be $2\pi l$, then we must have

$$g(y_3) - g(y_2) = lq, \qquad g(y_2) = lp/m \tag{3.39}$$

for some integers p and q. The integer m is fixed by the choice of KE_6^+ manifold: if $\mathcal{L}$ is the canonical line bundle, then m is the largest possible positive integer such that there exists a line bundle $\mathcal{N}$ with $\mathcal{L} = \mathcal{N}^m$. Furthermore, if the integers p and q are relatively prime Y_9 is simply connected. These conditions imply that we must choose

$$a = \frac{mq(2p+mq)}{(2p^2+2mpq+m^2q^2)} \tag{3.40}$$

and

$$l^2 = \frac{m^2(2p^2+2mpq+m^2q^2)}{2p^2(p+mq)^2} \tag{3.41}$$

Finally, we note that the four-form flux is given by (2.8) with

$$F = \frac{2^{3/2}}{3^{3/2}}\left[3y^2 dy\wedge dz - 8aJ_{\rm KE}\right] \tag{3.42}$$

It is straightforward to determine the additional conditions imposed by demanding that the four-form is properly quantised but we shall not do that here.

3.3 Non-compact AdS_2 solutions in $D = 11$ and defect CFTs

Given the solutions (3.36), we can return to the original angular variables ϕ and z and complete the squares in a different way, so the eleven-dimensional metric reads

$$\begin{aligned} ds^2 = \frac{2y^2}{3}ds^2(AdS_2) + \frac{32}{3}ds^2(KE_6^+) \\ + \frac{32}{3}\left[D\phi + \left(\frac{1}{2}-\frac{a}{4y}\right)dz\right]^2 + \frac{8y^2}{3q(y)}dy^2 + \frac{2q(y)}{3y^2}dz^2 \end{aligned} \tag{3.43}$$

Let us now consider letting the range of y be given by $y_4 \le y \le \infty$, where y_4 is the largest root of the quartic $q(y)$. Clearly this gives rise to non-compact solutions with AdS_2 factors. These are the analogue of the non-compact AdS_3 solutions of type IIB supergravity that were discussed in section 7 of [18].

Observe that when $a = 0$, after implementing the coordinate change $y^2 = 4\cosh^2 r$ and $\phi' = 4\phi + 2z$ we obtain

$$\tfrac{3}{8}ds^2 = \cosh^2 r ds^2(AdS_2) + dr^2 + \sinh^2 r dz^2 + 4\left[ds^2(KE_6^+) + \tfrac{1}{16}(d\phi' + 4B)^2\right] \tag{3.44}$$

This is simply the $AdS_4 \times SE_7$ solution of $D = 11$ supergravity where SE_7 is a seven-dimensional Sasaki-Einstein manifold. In particular, in the special case that we choose KE_6^+ to be $\mathbb{C}P^3$, we get the standard $AdS_4 \times S^7$ solution. Note that if SE_7 is regular or quasi-regular, then KE_6^+ is a globally defined manifold or orbifold, respectively, while if it is irregular, KE_6^+ is only locally defined.

We next observe that for general a, as $y \to \infty$ the solution behaves as if $a = 0$ and hence the solutions are all asymptotic to $AdS_4 \times SE_7$. By choosing the period of the coordinate z suitably, we can eliminate the potential conical singularity as y approaches y_4. With this period the non compact solutions are regular: they are fibrations of SE_7

over a four-dimensional space which is a warped product of AdS_2 with a disc parametrised by (y, z).

To interpret these solutions we consider for simplicity the case when $SE_7 = S^7$. Now, there are probe membranes in $AdS_4 \times S^7$ whose world-volume is $AdS_2 \times S^1$. More precisely, the AdS_2 world-volume is located in AdS_4 while the S^1 is the Hopf fibre of S^7. These configurations preserve 1/16-th of the Minkowski supersymmetry and are a generalisation[1] of those studied in [19] corresponding to defect CFTs. It is natural therefore to interpret our new solutions as the back reacted geometry of such probe branes. One might expect that the back reacted geometry of such branes to be localised in $\mathbb{C}P^3$, however, in our solutions the $\mathbb{C}P^3$ is still manifest. Hence our geometries seem to correspond to such probe membranes that have been "smeared" over the $\mathbb{C}P^3$.

We make a final observation about the $a = 1$ case, for which $q(y)$ has a double root at $y = 1$. By expanding the solution near $y = 1$ we find that the solution is asymptotically approaching the solutions discussed in section (6.15) below. In particular, for the special case when $KE_6^+ = \mathbb{C}P^3$, this is the solution found in [17] that describes the near horizon limit of membranes wrapping a holomorphic H^2/Γ in a Calabi-Yau five-fold. Thus, in this special case, our full non-compact solution, interpolates between $AdS_4 \times S^7$ and the solution of [17], while preserving an AdS_2 factor. Note that this is entirely analogous to the discussion of the non-compact type IIB AdS_3 solutions discussed in section 7 of [18].

4. Bubbles from fibrations over KE_{2n}^+ and Superstars

We will now use the same local Kähler metrics described at the beginning of section 3 to construct supersymmetric bubble solutions with S^3 factors in type IIB and S^2 factors in $D = 11$. The key point is simply to consider a different range of the variable x such that Ricci scalar R is now negative.

4.1 Type IIB solutions from fibrations over KE_4^+

We first observe that if we take $U(x) = 1 + a_1 x$, with $a_1 > 0$ to ensure that $R < 0$, and choose the four-dimensional Kähler-Einstein base, KE_4^+, to be $\mathbb{C}P^2$ we simply recover the $AdS_5 \times S^5$ solution. This becomes clear after making the coordinate transformation $\phi \to \phi - \frac{1}{2}t$. More generally by taking the same U but with different choices of $ds^2(KE_4^+)$ metric (note the corresponding leaves need not extend globally to form a compact Kähler-Einstein space) we can obtain an $AdS_5 \times SE_5$ solution, where SE_5 is any arbitrary five-dimensional Sasaki-Einstein manifold.

Let us now consider the solutions based on the more general cubic (3.17). Since taking $a_2 = 0$ returns to the $AdS_5 \times SE_5$ described above, we expect these solutions to correspond to excitations in the CFT dual of the Sasaki-Einstein solutions. We again must have $a_1 > 0$ to ensure $R < 0$. It is convenient to rescale the coordinate x so that $a_1 = 1$ (this leads to an overall scaling of the six-dimensional Kähler metric which can be absorbed into the

[1] In [20] probe membranes with world-volume $AdS_2 \times S^1$ were also considered but they are not the same as those being considered here as they preserve 1/4 of the supersymmetry.

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overall scale L of the $D = 10$ metric) giving

$$U = 1 + x + a_2 x^2 + \tfrac{1}{4} a_2^2 x^3. \tag{4.1}$$

If make the change of coordinate $x = 1/(r^2 + Q)$ where $Q = -a_2/2$ we find that the type IIB metric can be written as

$$ds^2 = -\tfrac{1}{4} H^{-2} f\, dt^2 + H\left[f^{-1} dr^2 + r^2 ds^2(S^3)\right] + ds^2(KE_4^+) + (D\phi + A)^2 \tag{4.2}$$

where

$$\begin{aligned} H &= 1 + \frac{Q}{r^2} \\ f &= 1 + r^2 H^3 \\ A &= \tfrac{1}{2} H^{-1} dt. \end{aligned} \tag{4.3}$$

When $KE_4^+ = \mathbb{C}P^2$ we see that this is precisely the single-charged AdS_5 "black hole" solution given in [21, 22], uplifted to $D = 10$ using an S^5, as described in [23]. The fact that we can replace the $\mathbb{C}P^2$ with any KE_4^+ is a consequence of the recent result that there is a consistent Kaluza-Klein truncation to minimal $D = 5$ gauge supergravity using any $D = 5$ Sasaki-Einstein space [24]. These $D = 10$ solutions were interpreted as corresponding to giant gravitons and were called superstars in [12].

4.2 $D = 11$ solutions from fibrations over KE_6^+

We now start with $U = 1 + a_2 x^2$ with $a_2 > 0$. If we choose $ds^2(KE_6^+)$ to be the metric on $\mathbb{C}P^3$ it is again easy to show that one recovers the $AdS_4 \times S^7$ solution. More generally we get $AdS_4 \times SE_7$ solutions for arbitrary Sasaki-Einstein seven-manifold SE_7 for suitable different choices of the local metric $ds^2(KE_6^+)$.

As before, solutions based on the more general quartic (3.29) should then correspond to excitations in the CFT dual of the Sasaki-Einstein solutions. Scaling x so that $a_2 = 1$, we have

$$U = 1 + x^2 + a_3 x^3 + \frac{a_3^2}{4} x^4 \tag{4.4}$$

If we make the change of variable $x = 1/(r + Q)$ where $Q = -a_3/2$ we find that the $D = 11$ metric can be written as

$$4^{2/3} ds^2 = -H^{-2} f dt^2 + H^2\left[f^{-1} dr^2 + r^2 ds^2(S^2)\right] + 4 ds^2(KE_6^+) + 4\left(D\phi + \tfrac{1}{2} A\right)^2 \tag{4.5}$$

where

$$\begin{aligned} H &= 1 + \frac{Q}{r} \\ f &= 1 + r^2 H^4 \\ A &= H^{-1} dt \end{aligned} \tag{4.6}$$

When $KE_4^+ = \mathbb{C}P^3$ this is precisely the supersymmetric single-charged AdS_4 "black hole" discussed in [25], uplifted to $D = 11$ using an S^7 as described in [23]. The fact that we can replace the $\mathbb{C}P^3$ with any KE_6^+ is very suggestive that there is a consistent Kaluza-Klein truncation to minimal $D = 4$ gauge supergravity using any $D = 7$ Sasaki-Einstein space (thus generalising the result of [24]; see also [26]). These $D = 11$ solutions were interpreted as corresponding to giant gravitons and were called superstars in [13].

5. New AdS solutions from multi-charged superstars

In this section we will derive new AdS solutions from the general three-charged and four-charged superstar solutions of type IIB and $D = 11$, respectively. The strategy is to first identify the Kähler geometry underlying the superstar solutions and then by a judicious rescaling and change of variables, construct a Kähler metric with positive Ricci scalar and use this to build the AdS solutions.

5.1 Type IIB three-charged superstars

We start by summarising the three-charge superstar geometry as presented in [23]. If we relate our time coordinate t to the time coordinate $\tilde{t}$ of that reference by $\tilde{t} = \frac{1}{2}t$, we find that the solution can be put in the bubble form (2.11) with

$$\begin{aligned} e^{4A} &= \mathcal{D}\mathcal{H}r^4 \\ P &= \frac{2}{r^2\mathcal{D}\mathcal{H}}\sum_i \mu_i^2 d\phi_i \\ ds_6^2 &= \frac{\mathcal{D}\mathcal{H}r^2}{f}dr^2 + r^2\sum_i H_i(d\mu_i^2 + \mu_i^2 d\phi^2) + \frac{1}{\mathcal{D}\mathcal{H}}\Big(\sum_i \mu_i^2 d\phi_i\Big)^2 \end{aligned} \tag{5.1}$$

where with $i = 1, 2, 3$

$$H_i = 1 + \frac{Q_i}{r^2} \tag{5.2}$$

and we have defined

$$\begin{aligned} \mathcal{H} &= H_1 H_2 H_3 \\ f &= 1 + r^2\mathcal{H} \\ \mathcal{D} &= \sum_i \frac{\mu_i^2}{H_i} \end{aligned} \tag{5.3}$$

Furthermore if we write the five-form, $\tilde{F}_5$ of [23] as $\tilde{F}_5 = 4F_5$, then we find[2] that it takes the form (2.12) and (2.13) with

$$F = \tfrac{1}{8}d\left[e^{4A}(dt + P)\right] + \tfrac{1}{2}J \tag{5.4}$$

where

$$J = -rdr \wedge \sum_i \mu_i^2 d\phi_i - \frac{r^2}{2}\sum_i H_i d(\mu_i^2) \wedge d\phi_i \tag{5.5}$$

It is easy to see that this form is closed. To see that it is indeed a Kähler form corresponding to the metric ds_6^2 it is convenient to choose the orthonormal frame as

$$\begin{aligned} e^i &= \frac{r\mathcal{H}^{1/2}}{f^{1/2}}\frac{\mu_i}{H_i^{1/2}}dr + rH_i^{1/2}d\mu_i \\ \tilde{e}^i &= \frac{C}{\mathcal{D}}\frac{\mu_i}{H_i^{1/2}}\sum_j \mu_j^2 d\phi_j + rH_i^{1/2}\mu_i d\phi_i \end{aligned} \tag{5.6}$$

[2]Note that there is a typo in the sign of the second term in equation (2.8) in [23].

One can then show that the metric can be written as

$$ds_6^2 = \sum_i \left(e^i \otimes e^i + \tilde{e}^i \otimes \tilde{e}^i\right) \tag{5.7}$$

provided that C satisfies

$$(C+r)^2 = r^2 + \mathcal{H}^{-1} = \frac{f}{\mathcal{H}} \tag{5.8}$$

In this frame, we find that J takes the conical form:

$$J = -\sum_i e^i \wedge \tilde{e}^i \tag{5.9}$$

It is possible to directly show that the relevant complex structure is integrable, which completes a direct confirmation that the metric ds_6^2 is indeed Kähler (of course this is all guaranteed since the solutions of [23] are known to preserve 1/8 supersymmetry). As a further check one can calculate the Ricci-form from the expression for P and, using the expression for J, the Ricci scalar. One can then compare with the expression for the warp factor and check that we have $R = -8e^{-4A}$.

5.2 Type IIB AdS_3 solutions

We now want to use the six-dimensional Kähler metrics coming from the three-charge superstars to construct new AdS_3 solutions. As stands it is not immediately obvious how to do so since these metrics have Ricci curvature $R < 0$ whereas for AdS_3 solutions we need $R > 0$. Recall that in the equal charge case the two types of solution arose from the same more general class of Kähler metrics with $U(x) = 1 + a_1 x + a_2 x^2 + (a_2^2/a_1)x^3$. These had $R = -8a_1 x^2$. Rescaling the coordinate x we could set either $a_1 = -1$ which led to AdS_3 solutions or $a_1 = 1$ leading to superstar solutions. Clearly if we want to use the three-charge superstar solutions to construct new AdS_3 geometries we need to extend the solutions by introducing the analogue of the a_1 parameter.

This is easy to do simply by using the scale invariance of solutions. The condition on the curvature (1.1) is clearly invariant under constant rescalings of the metric ds_{2n+2}^2 (as are of course the Kähler conditions). We know that the metric (5.1) is Kähler and satisfies (1.1) with Ricci scalar $R = -\mathcal{D}\mathcal{H}r^4$. Thus, from the rescaling symmetry, λds_6^2 is also a solution. Now consider making the change of variables $w = \lambda r^2$ and defining new parameters $q_i = \lambda Q_i$, so $H_i = 1 + q_i/w$. The rescaled metric can then be written as

$$ds_6^2 = \frac{Y}{4F}dw^2 + \sum_i (w+q_i)(d\mu_i^2 + \mu_i^2 d\phi_i^2) + \frac{F-1}{Y}\Big(\sum_i \mu_i^2 d\phi_i\Big)^2 \tag{5.10}$$

where we have introduced

$$\begin{aligned} Y(w) &= \sum_i \mu_i^2 (w+q_i)^{-1} \\ F(w) &= 1 + \lambda w^2 \prod_i (w+q_i)^{-1} \end{aligned} \tag{5.11}$$

while the scalar curvature and P are given by

$$\begin{aligned} R &= -\frac{8(F-1)}{w^2 Y} \\ P &= \frac{2(F-1)}{wY}\sum_i \mu_i^2 d\phi_i \end{aligned} \tag{5.12}$$

One notes the similarity in the parametrization with the form of the seven-dimensional Sasaki-Einstein metrics given in [14]. Note also that the new scale factor parameter λ appears only in $F(w)$.

The original superstar solution corresponded to $\lambda = 1$ with $R < 0$. For the AdS_3 we instead take $\lambda = -1$ with $R > 0$. For the metric to be positive definite we need to choose the range of w so that $w + q_i > 0$. This implies that $Y > 0$ and $F < 1$ so that $R > 0$ as required. Finally for the first term in (5.10) to be well-defined we also require $F \geq 0$. This can be achieved by choosing suitable values of $q_1 \leq q_2 \leq q_3$ so that the cubic $\prod(w+q_i) - w^2$ has three zeroes $w_1 < w_2 < w_3$ and demanding that $w_1 \leq w \leq w_2$ with $w_1 > -q_1$. Given that $F - 1 < 0$ it is not obvious that the metric is in fact positive definite. In the equal charge case, it is straightforward to show that it in fact is. Rather than show it for the general case, let us instead examine $ds^2(Y_7)$:

$$\begin{aligned} ds^2(Y_7) &= \tfrac{1}{4}(dz+P)^2 + e^{-4A} ds_6^2 \\ &= \frac{F}{4}dz^2 + \frac{1-F}{4w^2 F}dw^2 + \frac{1-F}{Yw^2}\sum_i (w+q_i)\left[d\mu_i^2 + \mu_i^2\Big(d\phi_i - \frac{w dz}{2(w+q_i)}\Big)^2\right] \end{aligned} \tag{5.13}$$

which is clearly positive definite.

To analyse the global structure of these metrics, we follow the approach of [14]. We first observe that the metrics are co-homogeneity three with $U(1)^4$ principle orbits which will degenerate at various points. The four local isometries are generated by ∂_z and ∂_{ϕ_i}. Globally we would like to find combinations of these Killing vectors which generate compact $U(1)$ orbits.

From the form (5.13) we see there are degenerations at $\mu_i = 0$ and also at $F = 0$. For the former, the Killing vector whose length is vanishing is simply ∂_{ϕ_i}. It is easy to see that for the metric to be smooth at $\mu_i = 0$ we require ϕ_i to have period 2π. For the degenerations at roots $w = w_1$ and $w = w_2$ of F the Killing vector whose length is vanishing is given by

$$l_i = c_i \partial_z + c_i \sum_j \frac{w_i}{2w_i + 2q_j}\partial_{\phi_j}, \tag{5.14}$$

for $i = 1, 2$ and some constant c_i. The requirement of regularity of the metric at these points can found either by requiring that l_i is normalised so that corresponding surface gravity κ_i is unity

$$\kappa_i^2 = \frac{g^{\mu\nu}\partial_\mu(l_i^2)\partial_\nu(l_i^2)}{4 l_i^2} \tag{5.15}$$

or by direct inspection of the metric by introducing a coordinate corresponding to l_i. In this case the latter is relatively straightforward and one finds that the constants c_i must

be given by

$$c_i^{-1} = -1 + \sum_j \frac{w_i}{2w_i + 2q_j} \tag{5.16}$$

which is again very similar to the conditions in [14].

We now have found conditions arising from five different degenerations: the three points $\mu_i = 0$ together with $w = w_1$ and $w = w_2$. However, there are only four isometries so there must be a relation of the form

$$pl_1 + ql_2 + \sum_j r_j \partial_{\phi_j} = 0 \tag{5.17}$$

for some co-prime integers (p, q, r_i). This then further restricts the parameters q_i. Since we can have $U(1)^2$ degenerations when $w = w_i$ and $\mu_j = 0$, we also require p and q are separately coprime to each of the r_i.

To ensure that we have a good solution of type IIB string theory we should also ensure that the five-form is suitably quantised. We will leave a detailed analysis of this issue for future work.

5.3 $D = 11$ four-charged superstars

Turning to solutions of $D = 11$ supergravity, we start by summarising the four-charged superstar geometry as an example of a 1/8 BPS state with an S^2 factor. In the next subsection we then adapt the metric as in the previous discussion to give a new class of AdS_2 solutions.

We first put the solution into our standard bubble form (2.14) starting from the form presented in [23]. We find, setting $g = \frac{1}{2}$ in [23], that

$$\begin{aligned} e^{3A} &= \mathcal{D}\mathcal{H}r^3 \\ P &= \frac{2}{r^2\mathcal{D}\mathcal{H}} \sum_i \mu_i^2 d\phi_i \\ ds_8^2 &= \frac{\mathcal{D}\mathcal{H}r}{f} dr^2 + 4r \sum_i H_i \left(d\mu_i^2 + \mu_i^2 d\phi_i^2\right) + \frac{4}{\mathcal{D}\mathcal{H}r}\Big(\sum_i \mu_i^2 d\phi_i\Big)^2 \end{aligned} \tag{5.18}$$

where for $i = 1, \ldots, 4$

$$H_i = 1 + \frac{Q_i}{r} \tag{5.19}$$

and we have defined

$$\begin{aligned} \mathcal{H} &= H_1 H_2 H_3 H_4 \\ f &= 1 + r^2\mathcal{H} \\ \mathcal{D} &= \sum_i \frac{\mu_i^2}{H_i} \end{aligned} \tag{5.20}$$

Furthermore the four-form flux[3] takes the form (2.16) with

$$F = d\left[e^{3A}(dt + P)\right] + J \tag{5.21}$$

[3]Note that there is a typo in the sign of the third term in equation (3.6) of [23].

where

$$J = -2dr \wedge \sum \mu_i^2 d\phi_i - 2r \sum H_i d(\mu_i^2) \wedge d\phi_i \tag{5.22}$$

which is clearly closed. To show that the metric ds_8^2 is indeed Kähler with Kähler form J, one can introduce an orthonormal frame in analogy with (5.6). Again, one could also check that the corresponding curvature satisfies (1.1) and that $R = -2e^{-3A}$.

5.4 $D = 11$ AdS_2 solutions

As before we can use the rescaling symmetry of the Kähler metric defining the four-charged superstar metric to obtain new $D = 11$ AdS_2 solutions. The internal space for the AdS_2 solutions we have already considered are analogous to the nine-dimensional Sasaki-Einstein spaces constructed in [9]. The new AdS_2 solutions we now consider are then analogous to the nine-dimensional Sasaki-Einstein spaces considered in [14].

We define $w = \lambda r$, rescale the metric by λ, define $q_i = \lambda Q_i$ and introduce as before

$$\begin{aligned} Y(w) &= \sum_i \mu_i^2 (w + q_i)^{-1} \\ F(w) &= 1 + \lambda^2 w^2 \prod (w + q_i)^{-1} \end{aligned} \tag{5.23}$$

We then find that the rescaled metric is given by

$$ds_8^2 = \frac{Y}{F} dw^2 + 4 \sum_i (w + q_i)(d\mu_i^2 + \mu_i^2 d\phi^2) + \frac{4(F-1)}{Y} \Big(\sum_i \mu_i^2 d\phi_i \Big)^2 \tag{5.24}$$

with

$$\begin{aligned} R &= -\frac{2(F-1)}{w^2 Y} \\ P &= \frac{2(F-1)}{wY} \sum_i \mu_i^2 d\phi_i \end{aligned}$$

In this form one immediately notes the similarity with the nine-dimensional Sasaki-Einstein metrics in [14]. Note also that although to derive this form we rescaled by λ, in the final expressions only λ^2 appears.

For the AdS_2 solutions we take $\lambda^2 = -1$, with $w + q_i > 0$ (which implies $Y > 0$ and $F < 1$ so $R > 0$) and $F \geq 0$. The metric on the internal manifold Y_9 can then be written as

$$\begin{aligned} ds^2(Y_9) &= (dz + P)^2 + e^{-3A} ds_8^2 \\ &= F dz^2 + \frac{1-F}{w^2 F} dw^2 + \frac{4(1-F)}{Y w^2} \sum_i (w + q_i) \Big[d\mu_i^2 + \mu_i^2 \Big(d\phi_i - \frac{w dz}{2(w + q_i)} \Big)^2 \Big] \end{aligned} \tag{5.25}$$

Again this is clearly positive definite. In analogy to the type IIB solution we choose $q_1 < q_2 < q_3 < q_4$ such that the quartic $\prod_i (w + q_i) - w^2$ has four roots $w_1 < w_2 < w_3 < w_4$ and require $w_2 \leq w \leq w_3$ with $w_3 > -q_1$.

The regularity conditions follow in analogy with the IIB solution, though now the principle orbits are $U(1)^5$. For regularity at $\mu_i = 0$ one is required to take ϕ_i to have period 2π. The vanishing norm Killing vectors at $w = w_2$ and $w = w_3$ are given by

$$l_i = c_i \partial_z + c_i \sum_j \frac{w_i}{2w_i + 2q_j} \partial_{\phi_j}, \tag{5.26}$$

for $i = 2, 3$ and some constant c_i. The requirement of regularity of the metric at these points then imposes as before

$$c_i^{-1} = \sum_j \frac{w_i}{2w_i + 2q_j} - 1. \tag{5.27}$$

We now have six different degenerations: the four points $\mu_i = 0$ together with $w = w_1$ and $w = w_2$ but only five isometries. Hence we require a relation of the form

$$p l_1 + q l_2 + \sum_j r_j \partial_{\phi_j} = 0 \tag{5.28}$$

for some co-prime integers (p, q, r_i). This then further restricts that parameters q_i. Since we can have $U(1)^2$ degenerations when $w = w_i$ and $\mu_j = 0$, we also require p and q are separately coprime to each of the r_i.

To ensure that we have a good solution of M-theory we should also ensure that the four-form is suitably quantised. We will leave a detailed analysis of this issue for future work.

6. Product of Kahler-Einstein spaces

We now turn to a different construction of Kähler metrics ds^2_{2n+2} satisfying (1.1). We will simply assume that it is locally the product of a set of two-dimensional Kähler-Einstein metrics

$$ds^2_{2n+2} = \sum_{i=1}^{n+1} ds^2(KE_2^{(i)}) \tag{6.1}$$

where $ds^2(KE_2^{(i)})$ is a two-dimensional Kähler-Einstein metric, i.e. locally proportional to the standard metric on S^2, T^2 or H^2. The Ricci form of ds^2_{2n+2} is given by

$$\mathcal{R} = \sum_{i=1}^{n+1} l_i J_i \tag{6.2}$$

where J_i are the Kähler forms of the $ds^2(KE_2^{(i)})$ metrics and l_i is zero, positive or negative depending on whether the metric is locally that on T^2, S^2 or H^2, respectively. We also have $P = \sum_i P_i$ with $dP_i = l_i J_i$ (no sum on i).

Globally, we will usually assume that ds^2_{2n+2} extends to the metric on a space M_{2n+2} which is simply a product of two-dimensional Kähler-Einstein spaces $M_{2n+2} = KE_2^{(1)} \times \cdots \times KE_2^{(n+1)}$. In the corresponding type IIB solutions ($n = 2$) and $D = 11$ solutions ($n = 3$),

one finds that the Killing spinors are independent of the coordinates on the $KE_2^{(i)}$. This means that the spaces $KE_2^{(i)}$ can be globally taken to be S^2, T^2, H^2 or a quotient H^2/Γ, the last giving a compact Riemann surface with genus greater than 1, while still preserving supersymmetry.

Note that in the special case that two of the l_i are equal, say $l_1 = l_2$, the analysis also covers the case when the product $KE_2^{(1)} \times KE_2^{(2)}$ is replaced with a more general four-dimensional Kähler-Einstein manifold, KE_4. Similar generalisations are possible if more of the l_i are equal.

Finally, in order to solve equation (1.1) we must impose

$$\sum_{i=1}^{n+1} l_i^2 = \left(\sum_{i=1}^{n+1} l_i\right)^2 \tag{6.3}$$

We also note that the Ricci scalar is given by

$$R = 2\sum_{i=1}^{n+1} l_i \tag{6.4}$$

6.1 Type IIB AdS_3 solutions

For the type IIB AdS_3 case we have $n = 2$. The warp factor is

$$e^{-4A} = \tfrac{1}{8}R = \tfrac{1}{4}\,(l_1 + l_2 + l_3) \tag{6.5}$$

and the two form F which determines the five-form flux via (2.3) is given by

$$F = \frac{1}{2(l_1 + l_2 + l_3)}\,[J_1(l_2 + l_3) + J_2(l_1 + l_3) + J_3(l_1 + l_2)] \tag{6.6}$$

The constraint (6.3) reads

$$l_1 l_2 + l_1 l_3 + l_2 l_3 = 0 \tag{6.7}$$

and we impose $R > 0$ to ensure that the warp factor is positive.

Let us analyse these constraints in more detail. Since we can permute the spaces KE_2^i we first order the parameters $l_1 \leq l_2 \leq l_3$. We then observe that a rescaling of the six-dimensional Kähler base space gives rise to the same $D = 10$ solution (up to rescaling of the overall factor L in the ten-dimensional metric). Since $R = 2(l_1 + l_2 + l_3) > 0$, we must have $l_3 > 0$ and hence we then rescale the metric ds_6^2 so that $l_3 = 1$. Solving (6.7) then gives $l_2 = -l_1/(l_1 + 1)$. Requiring $l_1 \leq l_2 \leq l_3$ gives a one parameter family of solutions specified by

$$(l_1, l_2, l_3) = (l_1, -\frac{l_1}{l_1 + 1}, 1) \tag{6.8}$$

with $l_1 \in [-1/2, 0]$.

Two equal l_i: it is interesting to look for special cases when two of the l_i are equal. As mentioned earlier, in this case we can generalise the solution by replacing the two identical KE_2 factors by KE_4. We find two cases. The first is when $(l_1, l_2, l_3) = (0, 0, 1)$ which gives $M_6 = T^4 \times S^2$. This leads to the well known $AdS_3 \times S^3 \times T^4$ solution corresponding to the near horizon geometry of two intersecting D3-branes.

The second and more interesting case is when $(l_1, l_2, l_3) = (-1/2, 1, 1)$ which gives $M_6 = H^2 \times KE_4^+$, where KE_4^+ is a positively curved Kähler-Einstein manifold. This means KE_4^+ is $S^2 \times S^2$, $\mathbb{C}P^2$ or a del Pezzo dP_k, $k = 3, \ldots, 8$. It is convenient to rescale the metric so that the H^2 factor has $l_1 = -1$ and hence $(l_1, l_2, l_3) = (-1, 2, 2)$. In the special case that $KE_4^+ = \mathbb{C}P^2$, this is a solution first found by Naka that describe D3-branes wrapping a holomorphic H^2 in a Calabi-Yau four-fold [16].

The more general solutions with arbitrary KE_4^+ were first given in [18]. Let us start by rewriting the solution in a standard form. Rescaling the metric $ds^2(KE_4^+)$ by a factor of three so that $\mathcal{R} = 6J_{\rm KE}$, the $D = 10$ solution then takes the form

$$\begin{aligned} \tfrac{\sqrt{3}}{2} ds^2 &= ds^2(AdS_3) + \tfrac{3}{4} ds^2(H^2) + \tfrac{9}{4} \left[ds^2(KE_4^+) + \tfrac{1}{9}(dz + P)^2 \right] \\ \tfrac{3}{4} F &= \left(-\tfrac{1}{4}\right) \left[-\tfrac{3}{2} J_{\rm KE} - 2\,\mathrm{vol}(H_2) \right] . \end{aligned} \tag{6.9}$$

The term in brackets in the first line is precisely the metric on a Sasaki-Einstein manifold, fibered over H^2 and with conventional normalization factors. To make the comparison with [18] we first note that the conventions for the flux differ by a factor of $-1/4$. Setting $L^2 = 2/\sqrt{3}$ in (2.1) and (2.3), and rescaling $ds^2(KE_4^+)$ again so that $\mathcal{R} = J_{\rm KE}$, we see that (6.9) is then exactly[4] the same as that in section 6.1 of [18] (with $d_3 = 0$). It was observed in [18] that one also obtains globally well defined solutions for (at least some) Sasaki-Einstein manifolds in the quasi-regular class, for which KE_4^+ is an orbifold.

6.2 Type IIB bubbles

For the corresponding type IIB bubble solutions the warp factor is given by

$$e^{-4A} = -\tfrac{1}{8} R = -\tfrac{1}{4} (l_1 + l_2 + l_3) \tag{6.10}$$

and the expression for the two-form determining the five-form flux via (2.12) is as in (6.6). For this case we need to impose (6.7) with $R < 0$ instead of $R > 0$. We find a one parameter family of solutions specified by

$$(l_1, l_2, l_3) = \left(-1, l_1, \frac{l_1}{l_1 - 1} \right) \tag{6.11}$$

with $l_1 \in [0, 1/2]$.

It is again interesting to look for special cases when two of the l_i are equal. We find two cases. The first is when $(l_1, l_2, l_3) = (-1, 0, 0)$ which gives $M_6 = T^4 \times H^2$. The second and more interesting case is when $(l_1, l_2, l_3) = (-1, -1, \frac{1}{2})$ which gives $M_6 = S^2 \times KE_4^-$. For example one could take KE_4^- to be the four-dimesnional Bergmann metric.

[4] There is a difference in the sign of term proportional to $J_{\rm KE}$ in the flux, but this corresponds to redefining $J_{\rm KE} \to -J_{\rm KE}$.

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6.3 $D = 11$ AdS_2 solutions

For the $D = 11$ case we have $n = 3$. For AdS_2 solutions the warp factor is given by

$$e^{-3A} = \tfrac{1}{2}R = l_1 + l_2 + l_3 + l_4 \tag{6.12}$$

with $R > 0$. The two-form that determines the four-form flux via (2.8) is given by

$$F = -\frac{J_1(l_2 + l_3 + l_4) + J_2(l_1 + l_3 + l_4) + J_3(l_1 + l_2 + l_4) + J_4(l_1 + l_2 + l_3)}{l_1 + l_2 + l_3 + l_4} \tag{6.13}$$

Assuming $l_1 \leq l_2 \leq l_3 \leq l_4$, $R > 0$ implies that $l_4 \geq 0$ and hence by rescaling we can take $l_4 = 1$. We find that there is now a two parameter family of solutions labeled by l_1, l_2 with

$$l_3 = -\frac{l_1 l_2 + l_1 + l_2}{l_1 + l_2 + 1}, \tag{6.14}$$

where the ranges of l_1 and l_2 are determined by the inequalities $l_1 \leq l_2 \leq l_3 \leq 1$.

Three equal l_i: first consider the case when three of the l_i are equal. One immediately finds that there are a just two possibilities: The first is when $(l_1, l_2, l_3, l_4) = (0, 0, 0, 1)$ corresponding to $M_8 = T^6 \times S^2$. This gives the well-known $AdS_2 \times S^3 \times T^6$ solution of $D = 11$ supergravity that arises as the near horizon limit of two intersecting membranes.

The second is when $(l_1, l_2, l_3, l_4) = (-1, 1, 1, 1)$ corresponding to $M_8 = KE_6^+ \times H_2$. In the special case that we take KE_6^+ to be a $\mathbb{C}P^3$ we recover the solution of $D = 11$ supergravity corresponding to the near horizon limit of a membrane wrapping a holomorphic H_2 embedded in a Calabi-Yau five-fold [17]. The existence of the more general solutions for arbitrary KE_6^+ was noted in a footnote in [18]. To put the solution in a standard form, we normalise the metric on KE_6^+ so that it has $\mathcal{R} = 8J_{\rm KE}$ so that the $D = 11$ solution then takes the form

$$\begin{aligned} 2^{2/3} ds^2 &= ds^2(AdS_2) + 2ds^2(H^2) + 16\left[ds^2(KE_6^+) + \tfrac{1}{16}(dz + P)^2\right] \\ 2F &= -\left[8J_{\rm KE} + 3\,\mathrm{vol}(H^2)\right]. \end{aligned} \tag{6.15}$$

Note that this has the same form as (6.9), with the terms in brackets in the first line giving a Sasaki-Einstein metric, fibered over H^2. In the special case that $KE_6^+ = \mathbb{C}P^3$ this agrees with the solution in [17].

Two equal l_i: we next consider the case when two of the l_i are equal. It is easier to take $l_1 = l_2$ and $l_3 \leq l_4$ instead of $l_1 \leq l_2 \leq l_3 \leq l_4$. We then note that if $l_1 = l_2 = 0$ we have $l_3 = 0$ and hence we have the $T^6 \times S^2$ solution discussed above. Otherwise we can always rescale so $l_1 = l_2 = \pm 1$. This leads to two one-parameter families of solutions

$$\begin{aligned} (l_1, l_2, l_3, l_4) &= \left(-1, -1, l_3, \frac{2l_3 - 1}{l_3 - 2}\right) && l_3 \in (2, 2 + \sqrt{3}] \\ (l_1, l_2, l_3, l_4) &= \left(1, 1, l_3, -\frac{2l_3 + 1}{l_3 + 2}\right) && l_3 \in (-2, -2 + \sqrt{3}]. \end{aligned} \tag{6.16}$$

Note that these also contain the interesting solution $(l_1, l_2, l_3, l_4) = (-1, -1, 2 + \sqrt{3}, 2 + \sqrt{3})$ corresponding to $M_8 = KE_4^- \times KE_4^+$.

6.4 $D = 11$ Bubbles

For the corresponding $D = 11$ bubble solutions the warp factor is given by

$$e^{-3A} = -\tfrac{1}{2}R = -(l_1 + l_2 + l_3 + l_4) \tag{6.17}$$

and the expression for the two-form determining the five-form flux via (2.15) is as in (6.13).

If we now impose $R < 0$ instead of $R > 0$, we find a two parameter family of solutions specified by

$$(l_1, l_2, l_3, l_4) = \left(-1, l_2, l_3, -\frac{l_1 l_2 - l_1 - l_2}{l_1 + l_2 - 1}\right) \tag{6.18}$$

with $-1 \leq l_2 \leq l_3 \leq l_4$.

There are then two possibilities when three of the l_i are equal: the first is when $(l_1, l_2, l_3, l_4) = (0, 0, 0, -1)$ corresponding to $M_8 = T^6 \times H^2$; the second is when $(l_1, l_2, l_3, l_4) = (-1, -1, -1, 1)$ corresponding to $M_8 = KE_6^- \times S^2$.

We next consider the case when two of the l_i are equal. We now find the one parameter family of solutions:

$$\begin{aligned} &\left(l_1, l_1, -\frac{l_1(l_1+2)}{2l_1+1}, 1\right), && l_1 \in [-2-\sqrt{3}, -1] \\ &\left(l_1, l_1, -\frac{l_1(l_1-2)}{2l_1-1}, -1\right), && l_1 \in (-1, 2-\sqrt{3}) \end{aligned} \tag{6.19}$$

In addition to the cases when three l_i are equal that we have already discussed, this family also contains the interesting solution $(-2-\sqrt{3}, -2-\sqrt{3}, 1, 1)$ corresponding to $M_8 = KE_4^- \times KE_4^+$.

7. Conclusion

It is remarkable that the equations for a generic supersymmetric warped $AdS_3 \times Y_7$ solution with F_5 flux in type IIB and for a generic supersymmetric warped $AdS_2 \times Y_9$ solution with electric flux in $D = 11$ supergravity are essentially the same [1, 2]. In each case the flux and local geometry of Y_{2n+3} is fixed by choosing a Kähler metric ds^2_{2n+2} satisfying (1.1).

Such backgrounds can arise from the near-horizon back-reacted geometry around D3- or M2-branes wrapped on a supersymmetric two-cycle. It is interesting to contrast these solutions with the $AdS_5 \times SE_5$ and $AdS_4 \times SE_7$ solutions, where SE_{2n+1} is a Sasaki-Einstein manifold, and which arise from unwrapped branes sitting at the apex of Ricci-flat Kähler cones. Again, locally, SE_{2n+1} is determined by a choice of Kähler metric $d\tilde{s}^2_{2n}$ which in this case is required to be Einstein. From this perspective, the construction of the wrapped brane solutions is very similar, except that the second-order tensorial Einstein condition is replaced by the fourth-order scalar condition (1.1) (together of course with flux which is fixed by ds^2_{2n+2}).

It was pointed out in [1, 2] that Kähler metrics satisfying (1.1) can also be used to construct supersymmetric bubble solutions. In this paper we have discussed three constructions of such Kähler metrics that give rise to new AdS and bubble solutions. The

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first construction, discussed in sections 3 and 4, is inspired by the construction of Sasaki-Einstein metrics in [8, 9]. In this case, the condition (1.1) could be integrated once, leaving a third-order nonlinear differential equation (3.11) for a single function $U(x)$. By restricting $U(x)$ to be polynomial, for type IIB we reproduced the solutions given in [11]. For $D = 11$ supergravity this led to a new one-parameter family of solutions. We also found new non-compact AdS_2 $D = 11$ solutions which can be interpreted as the duals of three-dimensional CFTs coupled to defects. It would be interesting to know whether or not there are interesting non-polynomial solutions to the differential equation (3.11).

The second construction of AdS solutions that we discussed in section 5 was found by elucidating the Kähler geometry underlying superstar solutions. These new AdS solutions, which generalise those of the first construction, are very analogous to the construction of SE metrics in [14]. Recall that these SE metric give rise to toric Ricci-flat Kähler cones. More generally, given that there are powerful techniques to study such toric cones, it will be interesting to try and adapt these techniques to study toric AdS_3 and AdS_2 solutions in the class of [1, 2].

It is interesting that the AdS_3 solutions of [11], that we recovered here in section 3, were also recently found from a different point of view in [27]. In that paper, an analysis of a general class of supersymmetric AdS black holes of minimal gauged supergravity in $D = 5$ was carried out. It seems likely that if one extended the analysis of [27] from minimal gauged supergravity to include two vector multiplets, that one would recover the new AdS_3 solutions of section 5. Extending the speculations of [27], it is natural to wonder if the solutions that we presented here in section 5 might describe the near horizon limit of an asymptotically $AdS_5 \times S^5$ black hole with horizon $S^1 \times Y_7$.

The third construction of solutions that we studied was to assume that ds^2_{2n+2} is locally a product of Kähler-Einstein metrics. This simple approach also leads to a rich class of AdS and bubble solutions.

In this paper we have focused on demonstrating that the metrics in the new AdS solutions are regular. It will be interesting to study the topology of the solutions and then determine the additional constraints on the parameters required to ensure that the fluxes are suitably quantised. It will then be straightforward to calculate the central charges of the dual SCFTs. Of particular interest, is to identify the CFTs dual to the new AdS solutions presented here. We expect that it will be most fruitful to focus on the type IIB AdS_3 solutions. The similarities of the construction of the type IIB AdS_3 solutions with $AdS_5 \times SE_5$ solutions is suggestive that the dual CFTs are also closely related.

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A. Type IIB AdS_3 from $D = 11$ AdS_2 solutions

Consider a general $D = 11$ solution of the form (2.7)-(2.9) with an eight-dimensional Kähler metric $ds^2(KE_8)$ which is locally the product of a six-dimensional Kähler metric and the flat metric on a torus

$$ds^2(KE_8) = ds^2(KE_6) + ds^2(T^2) \tag{A.1}$$

Assuming that globally in Y_9 the flat metric extends to the metric on a torus T^2, we can then dimensionally reduce to type IIA and then T-dualise to obtain a type IIB solution. Using the formulae in, for example, appendix C of [18], we deduce that the type IIB metric is given by

$$ds^2 = e^{3A/2}\left[ds^2(AdS_2) + (dx + A_1)^2 + (dz + P)^2 + e^{-3A}ds^2(M_6)\right] \tag{A.2}$$

with $dA_1 = -\operatorname{vol}(AdS_2)$. The first two-terms in the brackets are simply four times the metric on a unit radius AdS_3 and so after defining $e^{3A} = \frac{1}{4}e^{4A'}$ we can write this in the form

$$ds^2 = 2e^{2A'}\left[ds^2(AdS_3) + \tfrac{1}{4}(dz + P)^2 + e^{-4A'}ds_6^2\right] \tag{A.3}$$

This is exactly the form of (2.2) provided $L^2 = 2$. Similarly, using the conventions of [18], the five-form flux can be calculated and we find

$$-\tfrac{1}{4}F_5' = 4(1 + *)\operatorname{vol}(AdS_3) \wedge \left[-\tfrac{1}{8}(e^{4A'}\mathcal{R} - 4J) - \tfrac{1}{8}d(e^{4A'}) \wedge (dz + P)\right] \tag{A.4}$$

which, given $L^2 = 2$, agrees with (2.3) and (2.4) up to an overall difference in convention $F_5 = -\frac{1}{4}F_5'$.

References

[1] N. Kim, *AdS_3 solutions of IIB supergravity from D3-branes*, *JHEP* **01** (2006) 094 [hep-th/0511029].

[2] N. Kim and J.-D. Park, *Comments on AdS_2 solutions of $D = 11$ supergravity*, *JHEP* **09** (2006) 041 [hep-th/0607093].

[3] H. Lin, O. Lunin and J.M. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, *JHEP* **10** (2004) 025 [hep-th/0409174].

[4] A. Donos, *A description of 1/4 BPS configurations in minimal type-IIB SUGRA*, *Phys. Rev.* **D 75** (2007) 025010 [hep-th/0606199].

[5] A. Donos, *BPS states in type-IIB SUGRA with* $\mathrm{SO}(4) \times \mathrm{SO}(2)$ *(gauged) symmetry*, hep-th/0610259.

[6] E. Gava, G. Milanesi, K.S. Narain and M. O'Loughlin, 1/8 *BPS states in AdS/CFT*, hep-th/0611065.

[7] O.A.P. Mac Conamhna and E.O. Colgain, *Supersymmetric wrapped membranes, AdS_2 spaces and bubbling geometries*, hep-th/0612196.

[8] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Sasaki-Einstein metrics on $S(2) \times S(3)$*, *Adv. Theor. Math. Phys.* **8** (2004) 711 [hep-th/0403002].

[9] J.P. Gauntlett, D. Martelli, J.F. Sparks and D. Waldram, *A new infinite class of Sasaki-Einstein manifolds*, *Adv. Theor. Math. Phys.* **8** (2006) 987 [hep-th/0403038].

[10] D.N. Page and C.N. Pope, *Inhomogeneous Einstein metrics on complex line bundles*, *Class. and Quant. Grav.* **4** (1987) 213.

[11] J.P. Gauntlett, O.A.P. Mac Conamhna, T. Mateos and D. Waldram, *Supersymmetric AdS_3 solutions of type-IIB supergravity*, *Phys. Rev. Lett.* **97** (2006) 171601 [hep-th/0606221].

[12] R.C. Myers and O. Tafjord, *Superstars and giant gravitons*, *JHEP* **11** (2001) 009 [hep-th/0109127].

[13] F. Leblond, R.C. Myers and D.C. Page, *Superstars and giant gravitons in M-theory*, *JHEP* **01** (2002) 026 [hep-th/0111178].

[14] M. Cvetič, H. Lu, D.N. Page and C.N. Pope, *New Einstein-Sasaki spaces in five and higher dimensions*, *Phys. Rev. Lett.* **95** (2005) 071101 [hep-th/0504225].

[15] D. Martelli and J. Sparks, *Toric Sasaki-Einstein metrics on $S^2 \times S^3$*, *Phys. Lett.* **B 621** (2005) 208 [hep-th/0505027].

[16] M. Naka, *Various wrapped branes from gauged supergravities*, hep-th/0206141.

[17] J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, *Membranes wrapped on holomorphic curves*, *Phys. Rev.* **D 65** (2002) 026003 [hep-th/0105250].

[18] J.P. Gauntlett, O.A.P. Mac Conamhna, T. Mateos and D. Waldram, *New supersymmetric AdS_3 solutions*, *Phys. Rev.* **D 74** (2006) 106007 [hep-th/0608055].

[19] A. Karch and L. Randall, *Locally localized gravity*, *JHEP* **05** (2001) 008 [hep-th/0011156].

[20] K. Skenderis and M. Taylor, *Branes in AdS and pp-wave spacetimes*, *JHEP* **06** (2002) 025 [hep-th/0204054].

[21] L.A.J. London, *Arbitrary dimensional cosmological multi-black holes*, *Nucl. Phys.* **B 434** (1995) 709.

[22] K. Behrndt, A.H. Chamseddine and W.A. Sabra, *BPS black holes in $N = 2$ five dimensional AdS supergravity*, *Phys. Lett.* **B 442** (1998) 97 [hep-th/9807187].

[23] M. Cvetič et al., *Embedding AdS black holes in ten and eleven dimensions*, *Nucl. Phys.* **B 558** (1999) 96 [hep-th/9903214].

[24] A. Buchel and J.T. Liu, *Gauged supergravity from type-IIB string theory on $Y(p, q)$ manifolds*, hep-th/0608002.

[25] L.J. Romans, *Supersymmetric, cold and lukewarm black holes in cosmological Einstein-Maxwell theory*, *Nucl. Phys.* **B 383** (1992) 395 [hep-th/9203018].

[26] J.P. Gauntlett, E.O. Colgain and O. Varela, *Properties of some conformal field theories with M-theory duals*, *JHEP* **02** (2007) 049 [hep-th/0611219].

JHEP04(2007)005

[27] H.K. Kunduri, J. Lucietti and H.S. Reall, *Do supersymmetric anti-de Sitter black rings exist?*, *JHEP* **02** (2007) 026 [hep-th/0611351].